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Abstract

The Standard Solar Model (SSM) and its basic assumption of an equation of state of perfect gases for the plasmas of its interior is analyzed within the new theoretical framework of QED coherent states. We find that for $\frac{r}{r_{\odot}} \leq 0.3$ the SSM solution is unstable against collapse to a dense, *cold* coherent core and as a result a large suppression of B^8 -neutrinos is obtained, apparently solving the long standing solar neutrino problem.

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The Sun, so crucial to our human destiny, is but an average star, one rather anonymous among the 10^{20} or so that populate our Universe. And, as a typical representative of such huge population, being so close to us, it has been the object of extensive studies both experimental and theoretical such that most astrophysicists are today convinced that indeed we do know what is going on inside it. In this way a rather tight model of the Sun's interior has been built - the Standard Solar Model (SSM) - that accounts in a consistent fashion for the most diverse features of this average star, from its luminosity to the plethora of nuclear fusion reactions that are responsible for its long safe burning. All good and in order then, not quite: in the last thirty years *a small cloud has appeared in this serene, luminous sky*: the solar neutrino problem¹. But the faith in the SSM has in no way faltered, rather the puzzle is now generally believed to be solved by the old Pontecorvo idea of *neutrino mixing*, a subtle phenomenon that would reproduce in the lepton sector of the Standard Model of particle interactions the well established mixing of the quark sector. However, in spite of the remarkable effort that high energy physicists have made in the last decades to obtain a consistent mixing scheme, to this day no reliable solution is known, and the search continues. In this paper, instead, we shall try and submit to a critical evaluation the SSM itself with two main aims: first to assess whether the basic assumption of the equation of state of a perfect gas, that appears so natural is indeed so, and second whether a new consistent description of the Sun's interior exists that, leading to different basic parameters (pressure, temperature, density), solves the solar neutrino problem.

The framework in which such analysis shall be performed is that of *QED Coherence*, expounded at length in a recent book [2] that systematically describes a new class of solutions of QED dynamical equations in condensed matter, whose existence has been confirmed by several Authors, and in particular by the well known condensed matter theorist C.P. Enz [3].

¹For a nice, thorough account of this fascinating problem and for an extensive bibliography see the book of Ref [1]

As this new approach to condensed matter physics is not widely known, let us first review its main points concerning the plasmas of electrons, protons and several other nuclei that are expected to crowd the Sun's center. The general theory of coherent plasmas, described in Chap. 5 of Ref [3], leads to the following conclusions:

- (i) at zero temperature the gaseous state of each plasma, characterized by its plasma frequency ²

$$\omega_p = \frac{e}{m^{\frac{1}{2}}} \left(\frac{N}{V} \right)^{\frac{1}{2}} ; \quad (1)$$

is unstable against a transition to a coherent laser-like state, where all particles of the plasma oscillate in phase with a coherent electromagnetic field, which thus realizes an energy gain;

- (ii) the energy gain per particle, the gap Δ , is given by ($\rho = \left(\frac{N}{V} \right)$ is the number density)

$$\Delta = 2.95 e^2 \rho^{\frac{1}{3}} ; \quad (2)$$

- (iii) for $T \neq 0$, due to thermal fluctuations an incoherent plasma develops, which coexists with the coherent one, whose fraction F_i for a spin- $\frac{1}{2}$ fermion is given by

$$F_i = \frac{2}{\rho} \int_{p_F} \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E-E_F+\Delta}{T}\right) + 1} \simeq \frac{1}{\pi} \frac{mT}{\rho^{\frac{2}{3}}} \exp\left(-\frac{\Delta}{T}\right); \quad (3)$$

where p_F is the Fermi momentum and $E_F = \frac{p_F^2}{2m}$ is the Fermi energy;

- (iv) in order to determine at a given pressure the transition temperature, a detailed thermodynamical analysis ³ is necessary, however one notes that the gaseous state is certainly stable if F_i given by Eq.(3) is such that

$$F_i \geq 1 \quad (4)$$

²Throughout this paper we shall use the *natural units system* in which $\hbar = c = k = 1$

³See, for instance, for the case of water Chapt. 10 of Ref [2]

(v) for negligible F_i the equilibrium density is determined by

$$p = p_{Pauli} + p_c \quad ; \quad (5)$$

where p is the external pressure, p_{Pauli} is the *Pauli pressure* given by

$$p_{Pauli} = -\frac{\partial}{\partial V} \left(2V \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{p^2}{2m} \right)_N = \frac{2}{3} b \rho^{\frac{5}{3}} \quad \left(b = \frac{\pi^3}{10m} \right); \quad (6)$$

and p_c is the *coherent (negative) pressure*

$$p_c = -\frac{\partial}{\partial V} (N\Delta)_N = -\frac{a}{3} \rho^{\frac{4}{3}} \quad (a = 2.95e^2). \quad (7)$$

With these basic results let us turn to the Sun's parameters predicted by the SSM, reported in Table I. Our analysis, admittedly coarse and preliminary, will simply consist in computing for each solar radius r the gap Δ and the incoherent fraction F_i in terms of the temperature T and the density ρ and check whether the condition (4) is satisfied.

When this happens the gaseous state is stable and the evaluation of the SSM reliable. If on the other hand $F_i(r) \leq 1$, according to our theory (and our rough approximation) the basic assumption of the SSM fails, for the plasma finds it energetically favorable to collapse to the *Coherent Ground State* (CGS).

We emphasize that, though coarse, our analysis is basically correct, for the gaseous state comprises the exterior solar shell and the SSM solution *conquers the interior* starting from the known exterior structure through the basic equilibrium equations, and it will thus break down when the equation of state hypothesis breaks down. On the right columns of Table I we have reported our predictions for the gap Δ (Eq. (2)) and the incoherent fraction F_i (Eq. (3)) for the pressure and temperature of the SSM for the electrons' plasma.

Proceeding from the outside in, we see that the SSM is stable until $\frac{r}{r_\odot} \approx 0.3$, after which value the incoherent fraction becomes smaller than 1, and a condensed coherent phase gets formed. Thus we conclude that for $\frac{r}{r_\odot} \leq 0.3$ the SSM breaks down and a CGS appears. The question now is what are its features ?

The answer can immediately be found through the pressure equation (5). Taking the SSM value $p = p(0.3) = 9.3 \cdot 10^9$ bar, we find for the equilibrium density

$$\rho_{eq} = 1.29 \cdot 10^{27} cm^{-3} . \quad (8)$$

At this density the gap is

$$\Delta_{eq} = 5.9 \text{ keV} , \quad (9)$$

and $(F_i)_{eq}$ utterly negligible if we take the temperature T , as we must, equal to that predicted by the SSM at $\frac{r}{r_\odot} \simeq 0.3$, i.e.

$$T(0.3) = 0.55 \text{ keV} = 6.41 \cdot 10^6 \text{ K} . \quad (10)$$

We are now in a position to give a preliminary answer to the question of the title of this paper. According to QED at the center of the Sun there hides a coherent state of the electrons' plasma whose density and temperature are given by (8) and (10) respectively. On the other hand, at the temperature (10) the plasma of protons and of the other nuclei that are produced through the reactions of nuclear fusion are predicted to be in the gaseous state⁴. The size of such *coherent core* can be easily estimated by noting that according to the SSM it comprises 0.65 of the mass of the Sun, or a number of nucleons $N_N = 7.8 \cdot 10^{56}$, thus ($r_\odot = 7 \cdot 10^{10} \text{ cm}$)

$$\frac{r}{r_\odot} = \left[\frac{N_N}{\rho} \frac{3}{4\pi} \right]^{1/3} \frac{1}{r_\odot} = 0.075 . \quad (11)$$

One of the most important (preliminary) consequences of these uncanny results is upon the solar neutrino problem, that hinges mainly on the high energy neutrinos emerging from the β - decay of B^8 . The temperature dependence of their flux $\Phi(B^8)$ is very strong [1]:

$$\Phi(B^8) \approx \text{const} \cdot T^{18} , \quad (12)$$

⁴This can easily be appreciated by noting that their incoherent fractions exceed that of the electrons by the large mass ratios.

thus changing the central temperature from $T(0) = 1.54 \cdot 10^7 \text{ K}$ to the value (10) entails a suppression $\left(\frac{0.64}{1.54}\right)^{18} = 1.4 \cdot 10^{-7}$, even though some enhancement of the constant in (12) is to be expected due to the increased central density (see Table I and (8)). It has been remarked⁵ that a complete suppression of the B^8 -neutrinos would (a) solve the solar neutrino problem, and (b) have no consequence on the Sun's internal structure: the *coherent core* is thus seen to pass brilliantly this test. But how about the numerous hurdles that the SSM overcomes in an apparently natural way ? It is too soon to tell, a theoretical effort is needed comparable to the one carried out within the SSM, that we hope people will feel motivated to reproduce.

In conclusion, we have submitted the SSM and its equation of state to a critical analysis within the framework of *QED Coherence* [2]. We have found that for $\frac{r}{r_\odot} \leq 0.3$ the SSM is unstable against a phase transition towards the CGS of the electrons' plasma. As a result a dense, *cold coherent core* is predicted to arise characterized by Eqs (8)-(10), whose first recognizable and observable consequence is the suppression of B^8 -neutrinos, and the solution of the solar neutrino problem.

A last intriguing remark. In the formation of the *coherent core* a large energy is released:

$$E_c = N_N \left(\Delta - \frac{\pi^3}{10m} \cdot \rho^{2/3} \right) = 3.7 \cdot 10^{48} \text{ erg} \quad (13)$$

in a short time, the amount that the Sun at its present rate releases during 30 million years! It is a tantalizing thought that this monstrous explosion be at the origin of the solar system.

I wish to thank E. Del Giudice for interesting conversations.

⁵See the book of Ref. [1], pag.139 .

TABLES

| $r \backslash r_{\odot}$ | $m \backslash m_{\odot}$ | P (bar) | T (keV) | $\rho(cm^{-3})$ | $\Delta(keV)$ | F_i | $-p_c(bar)$ | $p_{Pauli}(bar)$ |
|--------------------------|--------------------------|---------------------|---------------------|----------------------|----------------------|-------|----------------------|----------------------|
| 0.00 | 0 | $2.4 \cdot 10^{11}$ | 1.327 | $9.18 \cdot 10^{25}$ | 2.66 | 0.35 | $1.1 \cdot 10^{11}$ | $4.9 \cdot 10^{10}$ |
| 0.06 | 0.10 | $1.9 \cdot 10^{11}$ | 1.246 | $6.64 \cdot 10^{25}$ | 2.39 | 0.44 | $7.7 \cdot 10^{10}$ | $2.55 \cdot 10^{10}$ |
| 0.12 | 0.12 | $1.1 \cdot 10^{11}$ | 1.034 | $4.34 \cdot 10^{25}$ | 2.07 | 0.45 | $4.4 \cdot 10^{10}$ | $1.4 \cdot 10^{10}$ |
| 0.16 | 0.23 | $7.0 \cdot 10^{10}$ | 0.902 | $3.03 \cdot 10^{25}$ | 1.84 | 0.48 | $2.69 \cdot 10^{10}$ | $7.7 \cdot 10^9$ |
| 0.20 | 0.34 | $4.4 \cdot 10^{10}$ | 0.798 | $2.12 \cdot 10^{25}$ | 1.63 | 0.54 | $1.68 \cdot 10^{10}$ | $4.3 \cdot 10^9$ |
| 0.26 | 0.53 | $1.8 \cdot 10^{10}$ | 0.641 | $1.07 \cdot 10^{25}$ | 1.30 | 0.69 | $6.75 \cdot 10^9$ | $1.4 \cdot 10^9$ |
| 0.31 | 0.65 | $9.3 \cdot 10^9$ | 0.552 | $6.36 \cdot 10^{24}$ | 1.09 | 0.89 | $3.37 \cdot 10^9$ | $5.7 \cdot 10^8$ |
| 0.43 | 0.83 | $1.8 \cdot 10^9$ | 0.398 | $1.72 \cdot 10^{24}$ | 0.71 | 1.85 | | |
| 0.49 | 0.89 | $7.7 \cdot 10^8$ | 0.306 | $8.58 \cdot 10^{23}$ | 0.56 | 2.84 | | |
| 0.58 | 0.94 | $2.3 \cdot 10^8$ | 0.265 | $3.21 \cdot 10^{23}$ | 0.40 | 4.97 | | |
| 0.68 | 0.97 | $7.1 \cdot 10^7$ | 0.205 | $1.30 \cdot 10^{23}$ | 0.30 | 7.36 | | |
| 0.83 | 0.99 | $8.2 \cdot 10^6$ | 0.094 | $3.35 \cdot 10^{22}$ | 0.19 | 4.76 | | |
| 0.91 | 0.999 | $1.2 \cdot 10^6$ | 0.059 | $1.04 \cdot 10^{22}$ | 0.13 | 5.44 | | |
| 1.00 | 1.00 | 0.12 | $4.9 \cdot 10^{-4}$ | $1.81 \cdot 10^{17}$ | $2.73 \cdot 10^{-3}$ | 2.55 | | |

TABLE I. The SSM predictions for mass, pressure, temperature and density at different depths. The last four columns report the predictions of *QED coherence* for the gap, the incoherent fraction, the coherent (negative) pressure and the Pauli pressure of the plasma of the electrons.

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